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ANALYSIS OF FILTERING AND CORRECTION OF THE POWER FACTOR IN DISTORTED BALANCE

FLORIN GABRIEL POPESCU[1], DRAGOS PASCULESCU[2], MARIUS MARCU[3], RAZVAN SLUSARIUC[4], GEORGETA BUICA[5], TEODORA LAZAR[6]

Abstract: The paper presents an analysis of the filtering and correction of the distorted steady state power factor. Due to the example analyzed, it was observed that in the distorted steady state, if the inductances and capacitances of the passive filters are properly sized, it is possible to obtain two additional effects in addition to the harmonic filtering for which they are used, namely the correction of the common power factor at 50 Hz and further reduction of the total current flowing through the network and of the total THD, which means a reduction of the distortion of the waveform of the current itself.

Keywords: Analysis, correction, filters, THD, power factor.

1. INTRODUCTION

Current applications for installation engineering frequently involve the presence of nonlinear loads that generate current harmonics, and therefore it may be necessary to correct the non-sinusoidal steady state power factor. When the presence of harmonics reaches an unacceptable level and consequently the adoption of LC filters is to be envisaged to compensate for one or more of them, the simultaneous suitability of such filters to correct the power factor at the fundamental frequency may be exploited: if properly sized, they can deliver all the required reactive power, thus avoiding the installation of dedicated capacitor banks [2], [7], [13].

Here are analysed and developed such operating conditions and the relevant sizing of the filter, also through an application example; to this end, a preliminary

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[1] *Ph.D., Lecturer Eng.,University of Petroşani, floringabriel@yahoo.com*
[2] *Ph.D., Associate Prof. Eng., University of Petroşani, pdragos_74@yahoo.com*
[3] *Ph.D., Associate Prof. Eng., University of Petroşani, mariusmarcu@upet.ro*
[4] *Ph.D., Lecturer Eng.,University of Petroşani, razvansir@yahoo.com*
[5] *Sc. Res. IInd, PhD.Eng., INCDPM "Alexandru Darabont" Bucharest, georgiana_buica@yahoo.com*
[6] *Ph.D., Asist. Eng., University of Petroşani, teomititica@yahoo.com*

introduction is made to some formulas and definitions of quantities useful for the analysis under consideration [4], [10].

2. QUANTITY ANALYSIS IN DISTORTED EQUILIBRIUM

A periodic, generally continuous and limited quantity can be developed in a Fourier series according to the following relation:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cdot \cos nx + b_n \cdot \sin nx \right) \tag{1}$$

where the first term of the right limb represents the average value of the function in period T, it is:

$$\frac{a_0}{2} = \frac{1}{T} \int_0^T f(x) \cdot dx \tag{2}$$

as the coefficients are calculated a_n and b_n of the series of:

$$a_n = \frac{2}{T} \int_0^T f(x) \cdot \cos nx \cdot dx \qquad b_n = \frac{2}{T} \int_0^T f(x) \cdot \cos nx \cdot dx \tag{3}$$

The Fourier series development can also be expressed in terms of cosine only as follows (in the temporal domain):

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} A_k \cdot \cos\left(k\omega t - \vartheta_k \right) \tag{4}$$

Switching from general quantities to alternating electrical quantities (average value zero $\frac{a_0}{2} = 0$) such as voltage and current, these, in a distorted state of equilibrium, can be expressed in the harmonic series with frequencies that are multiples of the fundamentals according to the following relations:

$$v = \sum_{k=1}^{\infty} \sqrt{2} \cdot V_k \cdot \cos\left(k\omega t - \vartheta_k \right) \qquad i = \sum_{k=1}^{\infty} \sqrt{2} \cdot I_k \cdot \cos\left(k\omega t - \vartheta_k - \varphi_k \right) \tag{5}$$

whose phase values RMS are defined as the square root of the sum of the squares of the values RMS of unique harmonics:

$$V = \sqrt{\sum_{k=1}^{\infty} V_k^2} \qquad I = \sqrt{\sum_{k=1}^{\infty} I_k^2} \tag{6}$$

In order to obtain information on the harmonic content of voltage and current waveforms and to take action if these values are high, the total harmonic distortion THD is defined:

$$THD_i = \frac{\sqrt{\sum_{k=2}^{\infty} I_k^2}}{I_1} \qquad THD_v = \frac{\sqrt{\sum_{k=2}^{\infty} V_k^2}}{V_1} \tag{7}$$

If $THD_i < 10\%$ and $THD_V < 5\%$, the harmonic ratio is considered low and thus no action will be taken, whereas in the opposite case one or more filters must be used for higher amplitude harmonics so that the values of the harmonic distortion ratios can be brought back to acceptable limits [1], [15], [18].

3. POWERS IN DISTORTED EQUILIBRIUM

In distorted equilibrium conditions, an extension of the sinusoidal steady-state powers is possible. In fact, the total apparent power S, the index of the thermal stress of an electrical component in a three-phase system, is defined as follows:

$$S = 3 \cdot V \cdot I = 3 \cdot \sqrt{\sum_{k=1}^{\infty} V_k^2 \cdot \sum_{k=1}^{\infty} I_k^2} \tag{8}$$

Given the presence of voltage and current harmonics added to the fundamental harmonic, the expressions for active power P and reactive power Q become:

$$P = 3 \cdot \sum_{k=1}^{\infty} V_k \cdot I_k \cdot \cos\varphi_k \qquad Q = 3 \cdot \sum_{k=1}^{\infty} V_k \cdot I_k \cdot \sin\varphi_k \tag{9}$$

Apparent power A is given by the following relation:

$$A = \sqrt{P^2 + Q^2} \tag{10}$$

This power differs from the total apparent power S defined in relation (9); in particular, the following relationship applies:

$$S^2 = P^2 + Q^2 + D^2 \tag{11}$$

where D is the *distortion* power and takes into account the distortion of voltage and current waveforms. The sum of the squares of the reactive power Q and the distortion power D gives the square of the *non-active power* N:

$$N^2 = Q^2 + D^2 \tag{12}$$

which is defined as "inactive" because it is given by the difference between the squares of the total apparent power S and the active power P:

$$N^2 = S^2 - P^2 \tag{13}$$

To explain this concept it is possible to give the graphical interpretation of figure 1, which is a three-dimensional extension of the two-dimensional triangle of power in the sinusoidal steady state. As can be seen, P, Q and D are the vertices of a parallelepiped whose main diagonal is S, A is the diagonal of the face having its edges P and Q, and N is the diagonal of the face whose edges are Q and D [6], [14], [17].

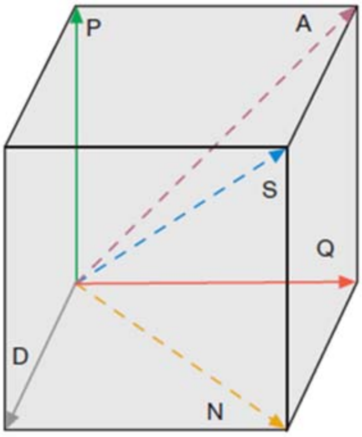

Fig.1. Three-dimensional extension of the two-dimensional triangle of power in the sinusoidal steady state

Along the supply line of a load operating with an active power P in a distorted steady state, the current defined in relation (6) flows with a voltage defined in the same formula; Consequently, the total phase shift factor ($\cos\phi$) between the active power P and the total apparent power S, occurring in the network is given by the relation:

$$\cos\phi = \frac{P}{S} \tag{14}$$

In the correction of the power factor, reference is made to this displacement factor by setting a target value of 0.9; thus, with the same value of the active power attracted by the load, the total apparent power (and, consequently, the current flowing) appeared in the network decreases [3], [11], [16]. The total displacement factor is an extension to the distorted equilibrium state of the normal power factor of the sinusoidal equilibrium state, which also results in this case:

$$\cos\varphi = \frac{P}{A} \tag{15}$$

If there were no distortions of the voltage and current waveforms, the factors appearing in the two equations above would coincide; on the contrary, in the presence of harmonics, they differ and the following relation is valid:

$$\cos\phi = \cos\varphi \cdot \cos\psi \tag{16}$$

in which the distortion factor $\cos\psi$ takes into account the presence of distortion power and is defined as:

$$\cos\psi = \frac{A}{S} \tag{17}$$

4. L-C FILTERS THAT FUNCTION AS CAPACITORS

Consider a branch of a passive L-C series filter resonating at a set frequency and graphically represent, as shown below, the capacitance and inductance of the reactance as a function of frequency, see figure 2.

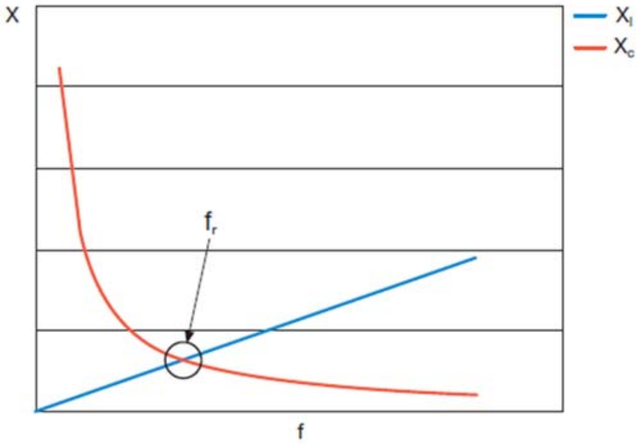

Fig.2. A branch of a passive L-C series filter resonating at a set frequency

As shown in the graph, it can be seen that below the resonant frequency $f_r = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$ capacitive reactance predominates and consequently the generated reactive power prevails over the drawn one:

$$Q = Q_L - Q_C = \omega \cdot L \cdot I^2 - \frac{1}{\omega \cdot C} \cdot I^2 < 0 \tag{18}$$

Therefore, by using passive filters for harmonic filtering at resonant frequencies, the correction of the power factor at lower frequencies is obtained, and this effect is taken into account for the sizing of the capacitive banks of the filters. In other words, when inducing LC filters, it is possible to choose such inductance and capacitance values simultaneously, so that the sum of the reactive power generated at the fundamental harmonic by all filters installed according to the reactive power required to make the total displacement factor observed from upstream network to reach 0.9 [8], [12].

At frequencies higher than the resonant one, the inductive effect predominates, but the amplitude of the harmonics present in the waveforms of the distorted current, in the common engineering applications of the installations, decreases as the frequency increases; as a result, the reactive power drawn by the filter at a frequency value higher than the resonance value decreases as the harmonic order increases, and in addition, for higher frequencies, the compensation bank presents itself to the whole network as an inductance, thus eliminating the possibility of resonance parallel to the inductance of the network [5], [9].

4.1. Case study

Suppose that a three-phase static rectifier Graetz in a fully controlled phase (figure 3) must be powered by a 50 Hz network with a short enough power to make it possible to ignore the distortion of the set of three voltages caused by the distorted current injected into the rectifier network.

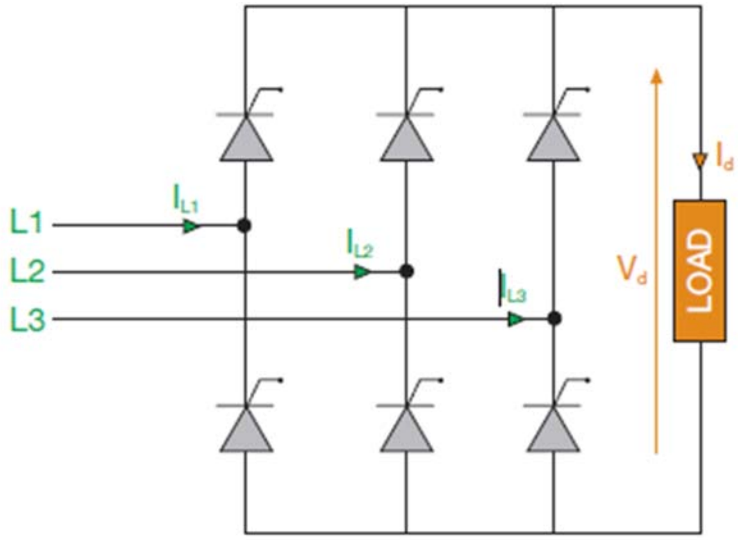

Fig.3. Graetz three-phase static rectifier in fully controlled phase

The current in each phase of the line (assuming a high inductance value on the DC side.) Has a rectangular waveform with the fundamental harmonic frequency equal to that of the sinusoidal voltage. The development in the Fourier series of such a waveform gives only harmonics of the order k = 6n ± 1 (n=0,1,2...), whose theoretical amplitude is inversely proportional to the harmonic of the order k, that is:

$$I_k = \frac{I_1}{k} \tag{19}$$

where I_1 is the amplitude of the fundamental harmonic (in the case examined equal to 50 Hz).

As, by the initial hypothesis, the voltage waveform is not distorted, its series development is reduced only to the fundamental harmonic and, consequently, the active and reactive powers absorbed by the rectifier (presumed without losses), calculated according to the relation (9), are equal to:

$$P = 3 \cdot \sum_{k=1}^{\infty} V_k \cdot I_k \cdot \cos\varphi_k = 3 \cdot V_1 \cdot I_1 \cdot \cos\varphi_1 = P = V_{d0} \cdot I_d \cdot \cos\alpha = P_d \qquad (20)$$

$$Q = 3 \cdot \sum_{k=1}^{\infty} V_k \cdot I_k \cdot \sin\varphi_k = 3 \cdot V_1 \cdot I_1 \cdot \sin\varphi_1 = 3 \cdot V_1 \cdot I_1 \cdot \sin\alpha = Q_1 \qquad (21)$$

V_{d0} is the value of the voltage on the DC side;

I_d is the value of the current in DC.

The apparent power that corresponds to these powers is:

$$A = \sqrt{P_1^2 + Q_1^2} = A_1 \qquad (22)$$

While the total apparent power observed at the power supply is:

$$S = 3 \cdot V \cdot I = 3 \cdot \sqrt{V_1^2 \cdot \sum_{k=1}^{\infty} I_k^2} \qquad (23)$$

a distortion power is present due to the distorted current waveform:

$$D = \sqrt{S^2 - A_1^2} \qquad (24)$$

Assuming that the bridge rectifier has a rated power, delivered on the DC side, equal to 140 kW, when it is powered by a grid with undistorted rated voltage and assuming that the switching is instantaneous and the phase control angle is such that $\cos\varphi = \cos\alpha = 0{,}8$, the following values are obtained for the powers on the AC side:

$$P = P_d = P_{do} \cdot \cos\alpha = 140 \cdot 0{,}8 = 112 \text{ [kW]} \qquad (25)$$

of which a first harmonic current:

$$I_1 = \frac{P}{\sqrt{3} \cdot U_n \cdot \cos\varphi} = \frac{112 \cdot 10^3}{\sqrt{3} \cdot 400 \cdot 0{,}8} = 202 \text{ [A]} \qquad (26)$$

and, consequently, reactive and apparent power is:

$$Q = \sqrt{3} \cdot U_n \cdot I_1 \cdot \sin\varphi = \sqrt{3} \cdot 400 \cdot 202 \cdot 0{,}6 = 84 \text{ [kVAr]} \tag{27}$$

$$A = \sqrt{P^2 + Q^2} = 140 \text{ [kVA]} \tag{28}$$

By developing in the Fourier series the distorted current waveform on the AC side, according to relation (19), the following values are obtained for the harmonic amplitudes (harmonics up to the 25th were taken into account):

Table 1. ***Values of harmonic amplitudes***

k	I_k **[A]**	I_k / I_1 **%**
1	202	100
5	40	20
7	29	14
11	18	9
13	15	8
17	12	6
19	11	5
23	9	4
25	8	4

Therefore, in the upstream network, in the absence of harmonic filters, a current would flow with a total RMS value equal to the square root of the square sum of RMS harmonic values given in the previous table:

$$I = \sqrt{\sum_{k=1}^{25} I_k^2} = 210 \text{ [A]} \tag{29}$$

with a total apparent power:

$$S = \sqrt{3} \cdot U_n \cdot I = \sqrt{3} \cdot 400 \cdot 210 = 146 \text{ [kVA]} \tag{30}$$

and a total harmonic distortion equal to:

$$THD = \frac{\sqrt{\sum_{k=5}^{25} I_k^2}}{I_1} = 29\% \tag{31}$$

Consequently, there would be a distortion factor $\cos\psi = \frac{A}{S} = 0{,}96$ and, seen in the upstream network, a phase shift factor $\cos\phi = \cos\varphi \cdot \cos\psi = 0{,}8 \cdot 0{,}96 = 0{,}77$.

The goal is to obtain a total phase shift factor equal to $\cos\phi' = 0,9$ and for this purpose it is assumed to dimension and introduce in parallel some L-C filters forthe 5th, -7th, 11th and 13th harmonics as shown in the figure 4:

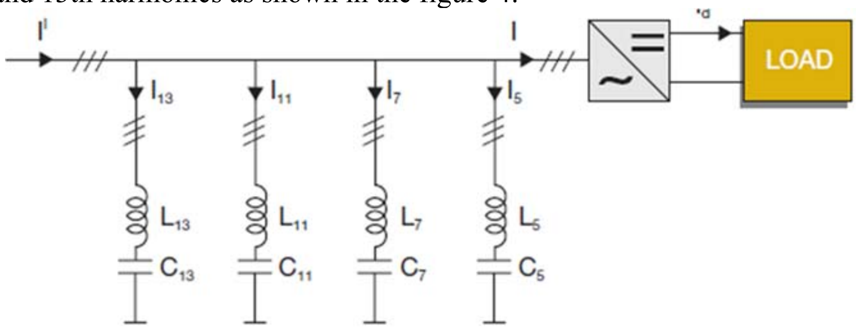

Fig.4. dimension and introduce in parallel some L-C filters

Therefore, the final value of $\cos\phi'$ must exceed 0.9. Assuming that this value is set to 0.91, the compensation of the reactive power obtained is equal to:

$$Q_c = P\cdot\left(tg\varphi - tg\varphi'\right) = 112\cdot\left(tg\left(\cos^{-1}(0,8) - tg(\cos^{-1}(0,91))\right)\right) = 33\,[\text{kVAr}] \qquad (32)$$

from which the final reactive power Q' after application of the power factor correction is:

$$Q' = Q - Q_c = 84 - 33 = 51\,[\text{kVAr}] \qquad (33)$$

By conducting tests and determining some inductance values for the harmonics to be filtered, the following capacitance values causing the series resonance are obtained:

$$C_k = \frac{1}{\left(2\pi f\right)^2\cdot L_k} \qquad (34)$$

Table 2. ***Inductance and capacitance values***

k	f[Hz]	L_k [mH]	C_k [μF]
5	250	1	406
7	350	2	103
11	550	1	84
13	650	1	6

The reactive power at 50 Hz provided, for example, by the L-C filter resonating at the 5th harmonic is calculated as follows:

$$I_{1,5} = \frac{U_n}{\sqrt{3}\left(2\pi \cdot 50 \cdot L_5 - \dfrac{1}{2\pi \cdot 50 \cdot C_5}\right)} \tag{35}$$

$$Q_{1,5} = 3\left(\frac{1}{2\pi \cdot 50 \cdot C_5} - 2\pi \cdot 50 \cdot L_5\right) \cdot I_{1,5}^2 \tag{36}$$

Similarly, the contributions of the other harmonics are calculated. The sum of the reactive compensation powers at 50 Hz is very close to the predefined one (with the values of inductance and capacitance shown in table 2); taking into account the value of the apparent power (at the same value of the active power absorbed P):

$$A' = \sqrt{P^2 + Q'^2} = 123 \text{ kVA} \tag{37}$$

the value of RMS of the first harmonic current becomes equal to:

$$I_1' = \frac{A'}{\sqrt{3} \cdot U_n} = \frac{123 \cdot 10^3}{\sqrt{3} \cdot 400} = 177 \text{ [A]} \tag{38}$$

which is approximately 12% lower than the initial value of a, at which the current values of the unfiltered harmonics correspond to:

Table 3. ***Unfiltered harmonic current values***

k	I_k **[A]**	I_k / I_1' **%**
17	10	6
19	9	5
23	8	4
25	7	4

As can be seen when comparing the absolute values of RMS (Root mean square) with the values in Tables 1 and 3, the correction of the power factor to 50 Hz, determines a reduction of the value of the square average of the first harmonic of the current, which leads to the reduction of unfiltered harmonics (because $I_k' = \frac{I_1'}{k}$).

This also implies a further reduction of the total current seen in the upstream network becoming I = 178 A (16% lower than the total initial current I) with a total apparent power S' :

$$S' = \sqrt{3} \cdot U_n \cdot I' = \sqrt{3} \cdot 400 \cdot 178 = 124 \text{ [kVA]} \tag{39}$$

The distortion factor goes from 0.96 to:

$$\cos\psi' = \frac{A'}{S'} = \frac{123}{124} = 0,99 \tag{40}$$

and the total displacement factor results:

$$\cos\phi' = \cos\varphi' \cdot \cos\psi' = 0,91 \cdot 0,99 = 0,906 \tag{41}$$

Thus, the established goal was achieved; otherwise, its set value should have been increased and the previous procedure should have been repeated. The total harmonic distortion ratio decreases to $THD' = 9,9\%$ (less than 10%).

5. CONCLUZII

In conclusion, thanks to this example, it has been observed that in the distorted steady state, if the inductances and capacitances of the passive filters are properly sized, it is possible to obtain two additional effects in addition to the harmonic filtering for which they are used:

- the correction of the common power factor at 50 Hz, because at the fundamental frequency the capacitive effect prevails over the inductive effect and, consequently, over the general reactive power over the absorbed one;
- by reducing, by correcting the power factor, the RMS the value of the fundamental harmonic of the current, consequently and RMS the values of unfiltered harmonics decrease; therefore, a further reduction of the total current flowing through the network and of the total THD is obtained, which means a reduction of the distortion of the current waveform itself.

REFERENCES

[1]. Buica G., Antonov A.E., Beiu C., Pasculescu D., Dobra R., *Occupational health and safety management in construction sector – the cost of work accidents*, Quality-Access to Success, Volume 18, Issue S1, pp. 35-40, 2017.

[2]. Dobra R., Buica G., Pasculescu D., Leba M., *Safety management diagnostic method regarding work cost accidents from electrical power installations*, Proceedings of the 1st International Conference on Industrial and Manufacturing Technologies, ISSN: 2227-4596, ISBN: 978-1-61804-186-9, 2013.

[3]. Marcu M.D., Popescu F.G., Pana L., *Modeling and simulation of power active filter for reducing harmonic pollution using the instantaneous reactive power theory*. Environmental Engineering and Management Journal, June 2014, Vol.13, No. 6, Pages: 1377-1382, DOI:10.30638/eemj.2014.148.

[4]. Niculescu T., Pasculescu D., Pana L., *Study of the operating states of intrinsic safety barriers of the electric equipment intended for use in atmospheres with explosion hazard*, WSEAS Transactions on Circuits and Systems, Volume 9, pp.430-439, 2010.

[5]. Niculescu T., Pasculescu D., Pasculescu V.M., Stoica I.O., „*Evaluation of electrical parameters of intrinsic safety barriers of the electrical equipment intended to be used in atmospheres with explosion hazard*”, 14th International Multidisciplinary Scientific Geoconference SGEM 2014, GeoConference on Informatics, Geoinformatics and Remote Sensing, Conference Proceedings, Volume I, Section Informatics, Albena, Bulgaria, 17-26 June,

2014, ISBN 978-619-7105-10-0, ISSN 1314-2704, DOI: 10.5593/SGEM2014/B21/S7.022, pag. 169-176.

[6]. Niculescu T., Arad V., Marcu M., Arad S., Popescu F.G., *Safety barrier of electrical equipment for environments with a potential explosion in underground coal mines*. MINING OF MINERAL DEPOSITS. Volume: 14 Issue: 3 Pages: 78-86, DOI: 10.33271/mining14.03.078, 2020.

[7]. Pasculescu D., Dobra R., Ahmad M.A., *Dosimetric Quantity System for Electromagnetic Fields Bio-effects*, International Journal of Scientific Research (IJSR) 5, no. 2 (2016): 28-32.

[8]. Pasculescu D., Niculescu T., Pana L., *Uses of Matlab Software to size intrinsic safety barriers of the electric equipment intended for use in atmospheres with explosion hazard*, Proceedings of the International Conference on Energy and Environment Technologies and Equipment (EEETE '10), RECENT ADVANCES in ENERGY and ENVIRONMENT TECHNOLOGIES and EQUIPMENT, Bucharest, Romania, Aprilie 20-22, 2010, ISSN: 1790-5095, ISBN: 978-960-474-181-6, pag. 17-21.

[9]. Popescu F.G., Arad S., Marcu M.D., Pana L., *Reducing energy consumption by modernizing drives of high capacity equipment used to extract lignite*, Papers SGEM2013/Conference Proceedings, Vol. Energy and clean technologies, pp. 183 - 190, Albena., Bulgaria, 2013.

[10]. Popescu F.G., Pasculescu D., Marcu M., Niculescu T., Handra A.D., *The technical and economic advantages of power factor correction*, Annals of University of Petrosani, Electrical Engineering, Vol. 21, pp.35-42, Petroşani, 2019.

[11]. Popescu F.G., Pasculescu D., Marcu M.D., Pasculescu V.M., *Analysis of current and voltage harmonics introduced by the drive systems of a bucket wheel excavator*, Mining of Mineral Deposits, Volume 14, Issue 4, 2020, Pages 40-46. ISSN: 24153435, DOI: 10.33271/mining14.04.040, 2020.

[12]. Popescu F.G., Păsculescu D., Păsculescu V.M., *Modern methods for analysis and reduction of current and voltage harmonics*, Editura LAP LAMBERT Academic Publishing, 2020, ISBN 978-620-0-56941-7, 233 pag.

[13]. Popescu F.G., Slusariuc R., Uţu I., *Maşini şi acţionări electrice -lucrări de laborator*, Editura Universitas, Petroşani, 2017.

[14]. Samoila B. L., Arad S., Marcu M. D., Popescu F. G., Utu I., *Contributions in modern electrical engineering higher education using dedicated applications*, International Symposium on Fundamentals of Electrical Engineering 2018, November 1 - 3, Bucharest, Romania, 2018.

[15]. Samoila, L., Arad, S., Petre, M., *Application for simulating the short-circuit current and the transient recovery voltage*, IEEE 16th International Conference on Environment and Electrical Engineering (EEEIC), pp. 2159-2163, Florence, Italy, 2016.

[16]. Samoila L., Utu I., *Măsurarea mărimilor electrice*. Universitas Publishing House, Petroşani, 2010.

[17]. Utu I., Marcu M., Niculescu T., Popescu F.G., Slusariuc R., *The use of semiconductor materials in the construction of modern equipment for adjustable drive of power equipment from a lignite open pit*, 18th International multidisciplinary Scientific Geoconference SGEM 2018 Conference Proceedings, Vol. 18- Informatics, Geoinformatics and Remote sensing, pp. 805-812, 2018.

[18]. Utu, I., Pasculescu, D., *Power Quality Study in Order to Comply with European Norms*. Supplement of "Quality - Access to Success" Journal, Vol.18, S1, January, pp. 366-371, 2017.